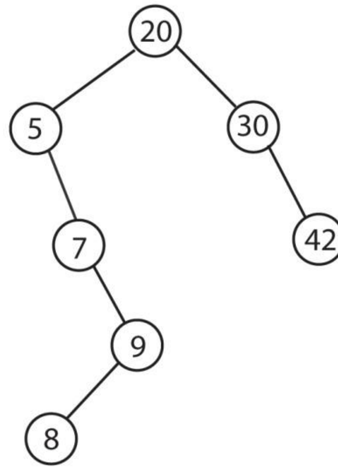
CS 151
Exam II Solutions

1. [15 points] Start with an empty Binary Search Tree. Draw a picture of the tree that will result from adding the following seven values to your tree, in the order they are given:

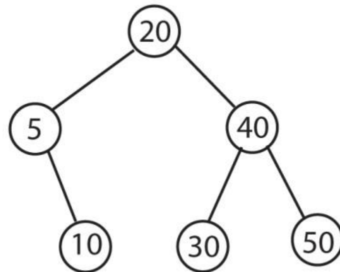
   20 30 5 7 42 9 8



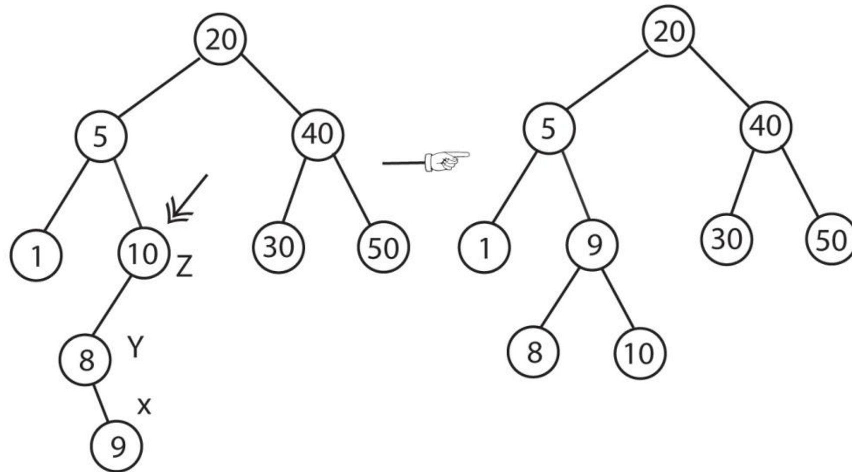
2. [5 points] What is the Balance Condition for AVL trees?

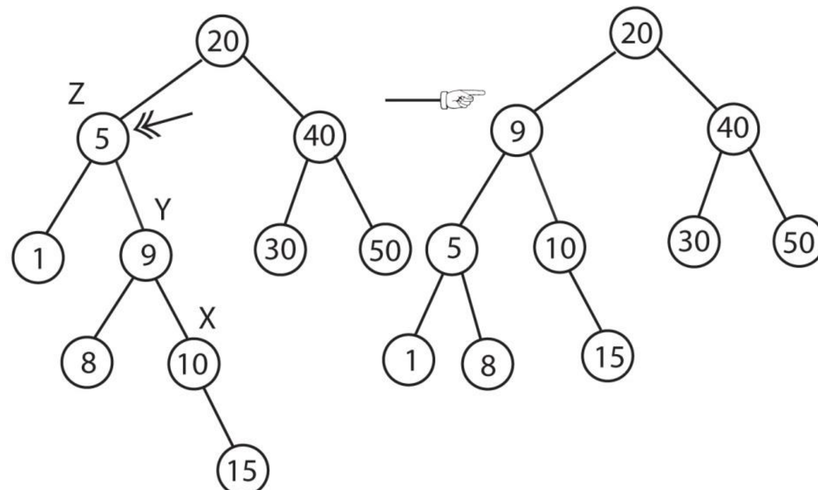
   At every node the heights of the left and right children differ by no more than 1.

3. [15 points] Start with the AVL tree shown below. Draw the AVL tree that will result from adding, in order, the following values: 1 8 9 15



You need to rebalance after every addition. 1 and 8 don't cause an imbalance. 9 does, then 15 does.:

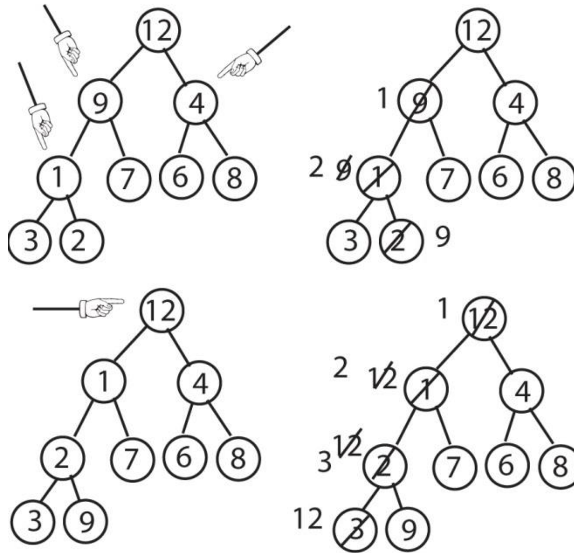




4. [15 points] Here is an array of 9 integer values:

12	9	4	1	7	6	8	3	2

We can think of this as a tree. Give the array that will result from turning this array into a heap using the linear-time algorithm we developed in class. You may find it easier to think in terms of trees, but put your final answer back into an array.



1	2	4	3	7	6	8	12	9

5. [20 points] For a text analysis project I need a map where the keys are words and the value associated with a word is the number of times that word appears in a text sample. So <”b ob”, 286> is a typical <key, value> pair. I could implement this as a hashmap with separate chaining (linked lists) you did in Lab 8, or as a hashmap with linear open addressing as we did in class, or as a treemap with an AVL tree. I will put n <key, value> pairs into this structure. The hashmaps use an array of size M.

   a) What are Big-Oh estimates of the worst-case and average case lookup times for a word that is in the map, for each of these structures.

	Average Case	Worst Case
Hashmap, chained	O(1)	O(n)
Hashmap, linear open address	O(1)	O(n)
Treemap	O(log(n))	O(log(n))

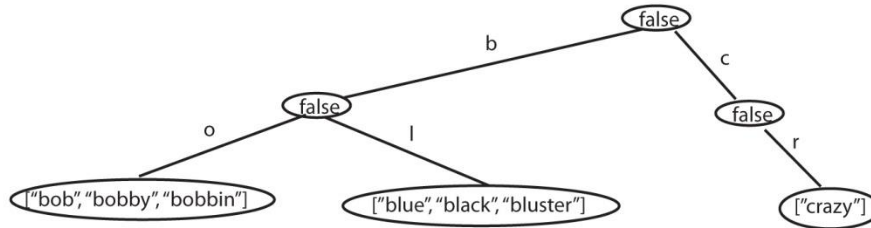
   b) Suppose I want to output the words in alphabetical order; which structure makes that easiest? Why?

   A treemap is by far the best structure for this. An inorder traversal of the tree puts the keys in increasing (and for strings, alphabetical) order.

   c) Suppose I want to output the words in order from the most frequent to the least frequent. Which structure makes that easiest? How would you do that?

   None of the structures makes this particularly easy. One solution would be to use a treemap. Traverse the tree, putting each <key, value> pair into an array; then sort the array on the values. A slightly faster solution would be to use an open-address hashmap, which already has the data in an array. Use a comparator that compares values and puts the open entries of the array at one end.

6. [15 points] As we discussed in class, Tries can use a lot of memory. One way around this is to use a trie for the first k levels (where k might be 3 or 4 or 5) and then have the leaf nodes store lists of all of the words with the same prefix. Here is such a structure with k==2 that stores “bob”, “bobby”, “bobbin”, “blue”, “black, “bluster” and “crazy”.



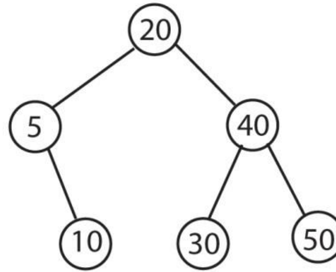
How would you create classes to make such a structure? Give class headers and class variables for such a trie. You don’t need to specify the methods, just the data variables. Don’t over-think this; I am just asking how to make a tree where the leaves have different data than the internal nodes.

```
abstract class MyTrie {
        ....
        // method signatures
}

class InternalTrieNode extends MyTrie {
        boolean isWord;
        MyTrie[ ] children;
        ...
        // method declarations
}

class LeafTrieNode extends MyTrie {
        ArrayList<String> words;
        ...
        // method declarations
}
```

7. [15 points] Give an algorithm (it is sufficient to explain it in English, but you can give pseudo-code if you wish) for the Binary Search Tree method **V findNext(K x)** which returns the value associated with the smallest key in the tree that is greater than x. If there is no key larger than x, return null. For example, with the following tree where the keys and values are the same



findNext(5) returns 10, findNext(18) returns 20, findNext(20) returns 30 and findNext(50) returns null. If the tree contains n nodes and is balanced (for example, if it is an AVL tree), estimate the running time for findNext(key).

```
V findNext(K x) {
    if (x < key) {
        if (left == null)
            return value;
        else {
            V t = left.findNext(x);
            if (t == null)
                return value;
            else
                return t;
        }
    }
    else if (x >=  key) {
        jf (right == null)
            return null;
        else
            return right.findNext(x);
    }
}
```